Four-Dimensional Torsional–Topological Spacetime Ontology

A Spacetime-Only Theory with Non-Orientable Defects,
Torsion-Domain-Wall Interfaces, and Defect Quantization

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Abstract

We formulate a spacetime-only theory in which the sole primitive is a four-dimensional differentiable manifold equipped with a Lorentzian metric and an affine connection. No fundamental matter fields are posited; "matter" and "gauge" phenomena arise as effective, emergent descriptions of geometry, topology, and torsion. Stable particle species are identified with compact defect worldtubes in spacetime, possibly non-orientable, classified by explicit topological invariants. Dynamics are governed by a metric-affine Einstein–Cartan theory extended by a propagating axial torsion mode with symmetry breaking. Interface mechanics arise as an effective derivative expansion of a torsion-sector domain wall localized on the defect boundary, rather than as primitive shell actions. The "curvature ledger" methodology is clarified as controlled 3+1 accounting identities justified by the compactness scaling $2GM/(Rc^2) \ll 1$ for hadronic and atomic defects. Three completion requirements are elevated to explicit theory obligations: a defect-cobordism quantization prescription, a derivation of effective couplings from the action with renormalization, and a rigorous obstruction-theoretic proof that admissible junctions enforce the Standard Model gauge structure.

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1 Introduction and Motivation

The program of describing matter as emergent from pure spacetime geometry has a distinguished history in theoretical physics. Wheeler's geometrodynamics [1, 2, 3] pursued the vision of "mass without mass" and "charge without charge," wherein all physical phenomena would arise from the topology and curvature of empty spacetime. This vision was captured in Wheeler's concept of *geons*—self-gravitating configurations of electromagnetic or gravitational radiation [4]—and the notion of *spacetime foam* at Planckian scales [5].

Einstein–Cartan theory, developed by Cartan [6, 7] and revived by Sciama [8] and Kibble [9], provides a natural extension of general relativity that incorporates torsion as a geometric degree of freedom coupled to intrinsic spin.[1] This framework offers a richer geometric arena for realizing Wheeler's program.

Simultaneously, the study of topological defects in cosmology and condensed matter—domain walls, cosmic strings, monopoles, and textures—has revealed how stable, particle-like structures can emerge from field configurations with nontrivial topology [11, 12, 13]. The Skyrme model [14, 15] demonstrated that baryons could be understood as topological solitons in a nonlinear sigma model, with the baryon number arising as a topological winding number [16, 17].

The present work synthesizes these traditions into a unified framework: a *spacetime-only theory* in which the primitive is the geometric-topological structure of a four-dimensional manifold, and all effective matter and gauge phenomena emerge from defects, torsion configurations, and their interactions. We draw upon the mathematical framework of topological quantum field theory (TQFT) [18, 19], cobordism theory [20, 21], and the classification of spinorial structures on manifolds [22, 23] to provide a rigorous foundation.

2 Kinematics: Primitive Structures and Admissibility

2.1 The Geometric Primitive

Definition 2.1 (Geometric Spacetime Primitive)**.** The primitive of the theory is a connected, four-dimensional differentiable manifold $\mathcal{M}$ equipped with:

(a) a Lorentzian metric g of signature $(-,+,+,+)$,

(b) an affine connection Γ not assumed torsion-free.

We denote the primitive as $(\mathcal{M}, g, \Gamma)$.

This structure defines a *Riemann–Cartan geometry*, the natural setting for Einstein–Cartan theory [10, 24]. The metric and connection are treated as independent variables, following the Palatini formulation [25].

Definition 2.2 (Torsion and Curvature)**.** Given the connection Γ, the torsion tensor and Riemann curvature tensor are defined by

$$T^{\lambda}{}_{\mu\nu} := \Gamma^{\lambda}_{\mu\nu} - \Gamma^{\lambda}_{\nu\mu}, \tag{1}$$

$$R^{\lambda}{}_{\rho\mu\nu}(\Gamma) := \partial_\mu \Gamma^{\lambda}_{\rho\nu} - \partial_\nu \Gamma^{\lambda}_{\rho\mu} + \Gamma^{\lambda}_{\sigma\mu}\Gamma^{\sigma}_{\rho\nu} - \Gamma^{\lambda}_{\sigma\nu}\Gamma^{\sigma}_{\rho\mu}. \tag{2}$$

Remark 2.1 (Levi–Civita Decomposition). The connection decomposes as $\Gamma = \mathring{\Gamma} + K$, where $\mathring{\Gamma}$ is the Levi–Civita connection of g and K is the *contorsion tensor*, expressible as a function of torsion [26]:

$$K^{\lambda}{}_{\mu\nu} = \frac{1}{2}\left(T^{\lambda}{}_{\mu\nu} + T_{\mu}{}^{\lambda}{}_{\nu} + T_{\nu}{}^{\lambda}{}_{\mu}\right). \tag{3}$$

The torsion-free limit of general relativity is recovered when $K \to 0$.

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[1] The theory is often referred to as Einstein–Cartan–Sciama–Kibble (ECSK) theory in recognition of these foundational contributions [10].

2.2 Topological Admissibility: Pin^+ Structure

To accommodate non-orientable defect sectors while maintaining consistent fermionic physics, we impose a topological admissibility condition.

Axiom 2.1 (Pin$^+$ Admissibility)**.** The manifold $\mathcal{M}$ admits a Pin$^+$ structure, enabling consistent non-orientable defect sectors without introducing fundamental spinor matter fields.

The Pin$^\pm$ groups are the double covers of the orthogonal group $O(n)$, generalizing the Spin group to non-orientable settings [22, 27]. For a manifold to admit a Pin$^+$ structure, the obstruction class $w_2 \in H^2(\mathcal{M}; \mathbb{Z}_2)$ must vanish, while Pin$^-$ structures require $w_2 + w_1^2 = 0$, where w_i denotes the i-th Stiefel–Whitney class [21].[2]

3 Foundational Axioms and Physical Interpretation

Axiom 3.1 (Geometric Monism)**.** There are no fundamental "matter fields" or "gauge fields" external to $(\mathcal{M}, g, \Gamma)$. All effective matter, charge, and gauge phenomena are emergent descriptions of geometric and topological structures of $\mathcal{M}$.

This axiom embodies Wheeler's vision of "physics as geometry" [2]. It represents a strong form of geometric unification, more radical than Kaluza–Klein theories which introduce additional dimensions [28, 29], or than Einstein's later unified field theories which introduced additional tensor fields [30, 31].

Principle 3.1 (Defect Ontology)**.** Stable particle species correspond to stable equivalence classes of compact worldtubes $W \subset \mathcal{M}$ (possibly non-orientable), classified by explicitly specified topological invariants $\mathcal{I}(W)$.

This principle draws upon the rich theory of topological defects in field theory and cosmology [11, 13, 32]. The key insight is that topological protection can stabilize extended objects against decay, just as the baryon number in the Skyrme model arises from the homotopy class $\pi_3(S^3) \cong \mathbb{Z}$ [15, 16].

Principle 3.2 (Emergent Interface Properties)**.** Interface properties (surface tension, bending rigidity) are not primitives. They arise as effective coefficients after integrating torsion dynamics across a thin torsion-domain-wall layer localized on ∂W.

This principle distinguishes the present theory from approaches that postulate fundamental brane or membrane actions [58, 59, 60]. Instead, all interface dynamics emerge from the bulk torsion sector through a derivative expansion, analogous to how effective membrane actions arise in the thin-wall limit of domain wall solutions in scalar field theory [37].

Principle 3.3 (Effective Equivalence Principle)**.** In the long-wavelength limit of weak torsion gradients and coarse-grained defect structure, the center-of-mass motion of a defect follows an effective geodesic principle. Spin/chirality-dependent deviations arise as controlled corrections from torsion coupling and multipole structure.

This principle connects to extensive literature on the motion of spinning particles in curved spacetime [47, 48] and torsion-induced corrections to geodesic motion in Einstein–Cartan theory [10, 49].

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[2] The choice of Pin$^+$ over Pin$^-$ has physical implications for the transformation properties of fermions under orientation-reversing diffeomorphisms; see [22] for a detailed discussion.

4 Dynamics: Unified Bulk Action

4.1 Total Bulk Action

Postulate 4.1 (Unified Bulk Action)**.** The fundamental action contains only bulk geometric terms:

$$S_{\text{bulk}}[g, \Gamma, S] = \int_{\mathcal{M}} d^4x \sqrt{-g} \left[\frac{1}{2\kappa} R(\Gamma) + \mathcal{L}_{\text{tors}}(S^\mu; g, \Gamma) \right], \tag{4}$$

where $\kappa := 8\pi G/c^4$ and S^μ denotes an axial torsion mode defined below.

The first term is the Einstein–Cartan–Palatini action, which reduces to the Einstein–Hilbert action in the torsion-free limit [25, 10]. The second term governs the dynamics of a propagating torsion mode.

4.2 Axial Torsion Sector

Definition 4.1 (Axial Torsion Mode)**.** Define an axial vector S^μ from torsion by the totally antisymmetric component:

$$S^\mu \propto \varepsilon^{\mu\nu\rho\sigma} T_{\nu\rho\sigma}. \tag{5}$$

In the effective theory, S^μ is treated as a dynamical axial torsion degree of freedom.

This decomposition follows the standard irreducible decomposition of torsion into vector, axial-vector, and tensor components [10, 39]. The axial component S^μ is particularly significant as it couples to the spin current of Dirac fermions [35].

Postulate 4.2 (Einstein–Cartan–Proca Torsion Lagrangian)**.** The torsion sector Lagrangian is:

$$\mathcal{L}_{\text{tors}} = \frac{\beta}{2} \nabla_{[\mu} S_{\nu]} \nabla^{[\mu} S^{\nu]} - \frac{\lambda}{4} \left(S_\mu S^\mu - S_0^2 \right)^2 - \Lambda_{\text{vac}}, \tag{6}$$

where $\beta > 0$ sets the kinetic normalization, $\lambda > 0$ the stiffness/self-coupling, S_0 the symmetry-breaking scale, and Λ_{vac} is a vacuum-energy subtraction fixing the reference energy of the torsion vacuum.

This structure is analogous to the Abelian Higgs model that produces Nielsen–Olesen vortex strings [36], but applied to the torsion sector. The Mexican-hat potential with vacuum expectation value $|S| = S_0$ allows for domain wall solutions interpolating between different torsion vacua.

Remark 4.1 (Vacuum Subtraction). For a monist theory, the vacuum is a torsionful medium rather than "empty" Minkowski space. The vacuum subtraction Λ_{vac} is required to define defect energies relative to the chosen torsion ground state and to avoid double-counting vacuum energy in effective surface tensions. This is analogous to the renormalization of the cosmological constant in quantum field theory [38].

5 Defects as Boundary Conditions and Topological Sectors

5.1 Defect Worldtubes

Definition 5.1 (Defect Worldtube)**.** A *defect* is a compact (possibly non-orientable) 4D submanifold-with-boundary $W \subset \mathcal{M}$ representing a particle history. Its *interface* is the hypersurface $\Sigma := \partial W$ when an excision/interface description is employed.

This definition generalizes the worldline of a point particle to an extended object with internal structure. The boundary Σ is a timelike hypersurface that separates the defect interior from the exterior bulk spacetime.

Definition 5.2 (Defect Sector Specification)**.** A *defect sector* is specified by:

(i) a topological invariant package $\mathcal{I}(W)$ (e.g., $\pi_1(W)$, orientability class, junction data),

(ii) boundary conditions on (g, Γ, S) on and near Σ consistent with $\mathcal{I}(W)$,

(iii) (optionally) bundle/holonomy data on $\mathcal{M} \setminus W$ for effective gauge structures.

The topological invariants classify defects into discrete sectors, analogous to how homotopy groups classify topological defects in field theory [33, 34]. The fundamental group $\pi_1(W)$ is particularly important for detecting non-simply-connected defect topologies.

5.2 Canonical Worldtube Types

The following postulates specify the topological structure of defect worldtubes corresponding to known particle species. These assignments should be understood as the *simplest* topological realizations consistent with observed quantum numbers.

Postulate 5.1 (Leptons)**.** A lepton worldtube is modeled by a punctured $\mathbb{RP}^4$-type defect:

$$W_{\mathbb{RP}} \simeq \mathbb{RP}^4 \setminus \{\text{point}\}, \quad \pi_1(W_{\mathbb{RP}}) \cong \mathbb{Z}_2. \tag{7}$$

The non-orientability of $\mathbb{RP}^4$ (which has $w_1 \neq 0$) encodes the fermionic nature of the lepton, while the $\mathbb{Z}_2$ fundamental group reflects the discrete symmetry associated with lepton number.

Postulate 5.2 (Quarks)**.** A quark worldtube is modeled as a mapping torus of an orientation-reversing diffeomorphism of S^3:

$$W_{\text{rev}} := S^3 \times [0,1]/(x,1) \sim (\phi(x),0), \quad \pi_1(W_{\text{rev}}) \cong \mathbb{Z}. \tag{8}$$

The $\mathbb{Z}$ fundamental group of the quark worldtube allows for fractional quantum numbers when combined with junction constraints (see below).

Postulate 5.3 (Baryons: Minimal Package)**.** A baryon is modeled by:

(a) a minimal degree-$B = 1$ topological sector (Skyrmion-like, interpreted as an effective coordinate on defect topology rather than a fundamental matter field),

(b) a tripartite junction constraint on Σ producing $\mathbb{Z}_3$ center holonomies z_i obeying $z_1 z_2 z_3 = 1$.

The Skyrmion language [14, 15, 16] is retained as a minimal effective parameterization. The tripartite junction with $\mathbb{Z}_3$ constraint captures the essential color structure of baryons as *qqq* composites.[3]

Remark 5.1 (Strict Monism and Skyrmions). In a fully closed monist theory, one must ultimately express the baryon sector purely in terms of connection, holonomy, and torsion topology rather than introducing an independent $U(x) \in \mathrm{SU}(2)$ field as fundamental content. The Skyrmion description is an effective parameterization, justified by Witten's demonstration that baryons emerge as solitons in the large-N_c limit of QCD [40, 17].

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[3] In QCD, the baryon wave function must be a color singlet, requiring the product of three color charges (each transforming in the fundamental representation of SU(3)) to be trivial. The $\mathbb{Z}_3$ center constraint is the topological shadow of this requirement.

6 Interfaces from Torsion Domain Walls

6.1 Thin-Wall Ansatz and Emergent Surface Tension

Construction 6.1 (Torsion-Domain-Wall Interface)**.** Let Σ be a timelike hypersurface separating two regions (defect interior/exterior) distinguished by torsion-sector order parameters. Introduce Gaussian normal coordinates in a neighborhood of Σ with transverse coordinate z (so $z = 0$ on Σ), and consider torsion configurations of the form

$$S^{\mu}(x) \approx \bar{S}^{\mu}(z;\xi), \tag{9}$$

where ξ are coordinates on Σ and $\bar{S}^{\mu}$ interpolates between vacua as $z \to \pm\infty$.

This construction follows the standard thin-wall approximation for domain walls in field theory [37, 13].

Definition 6.1 (Emergent Surface Tension)**.** Define the surface tension σ as the energy per unit area of the torsion-domain-wall profile, relative to vacuum:

$$\sigma[\bar{S}] := \int_{-\infty}^{+\infty} dz \left[\frac{\beta}{2}\left(\partial_z \bar{S}^{\mu}\right)\left(\partial_z \bar{S}_{\mu}\right) + \frac{\lambda}{4}\left(\bar{S}^{\mu}\bar{S}_{\mu} - S_0^2\right)^2\right]. \tag{10}$$

Remark 6.1 (Monism Restored). Equation (10) makes σ *derived* from the torsion sector parameters (β, λ, S_0) and the chosen defect boundary conditions. No primitive material constant is introduced, in accordance with Principle 3.2.

6.2 Bending Rigidity from Derivative Expansion

A curved interface forces slow variation of the wall profile along Σ and produces higher-derivative corrections. The standard effective-field-theory procedure [41, 42] yields an induced worldvolume action:

$$S_{\Sigma}^{\text{eff}} = \int_{\Sigma} d^3\xi \sqrt{-\gamma}\left[\sigma + \frac{\kappa_b}{2}(2H)^2 + \bar{\kappa} K_G + \cdots\right], \tag{11}$$

where γ is the induced metric on Σ, H is the mean curvature, and K_G is the Gaussian curvature of spatial sections.[4]

Definition 6.2 (Emergent Bending Modulus)**.** In the thin-wall regime, the bending rigidity κ_b is fixed by moments of the transverse energy density:

$$\kappa_b[\bar{S}] \sim \int_{-\infty}^{+\infty} dz\, z^2 \left[\frac{\beta}{2}\left(\partial_z \bar{S}^{\mu}\right)\left(\partial_z \bar{S}_{\mu}\right) + \frac{\lambda}{4}\left(\bar{S}^{\mu}\bar{S}_{\mu} - S_0^2\right)^2\right], \tag{12}$$

up to normalization fixed by the precise matching convention.

Equations (11)–(12) implement the requested correction: the "shell action" is not fundamental; it is an emergent effective description of a torsion-domain-wall interface.

7 Curvature, Backreaction, and Counter-Curvature Balance

7.1 Metric-Affine Hamiltonian Constraint

Let Σ_t be a spacelike foliation with induced metric h_{ij}, unit normal n^{μ}, and extrinsic curvature K_{ij}. In a metric-affine theory, the Hamiltonian constraint takes the schematic form [10, 45]:

$$R^{(3)} + K^2 - K_{ij}K^{ij} = \frac{16\pi G}{c^4} E_{\text{eff}} + Q_{\text{tors}}(h, K, S, \nabla S), \tag{13}$$

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[4] This is the Helfrich–Canham form familiar from membrane physics [43, 44], here derived rather than postulated.

where $E_{\text{eff}} := T^{\text{eff}}_{\mu\nu} n^\mu n^\nu$ is the effective energy density derived from the bulk torsion sector and defect boundary conditions, and Q_{tors} collects additional torsion/contorsion contributions.

The terms $K^2 - K_{ij}K^{ij}$ and Q_{tors} are precisely the *counter-curvature channels* that can balance the curvature implied by E_{eff}, preventing singular collapse in high-density regimes—the same structural mechanism exploited in torsion-bounce cosmology [45, 46].

7.2 Curvature Ledger as Controlled Bookkeeping

Construction 7.1 (Curvature Ledger Map)**.** On time-symmetric slices ($K_{ij} = 0$) and in regimes where Q_{tors} is negligible in the chosen gauge, define the ledger entry as:

$$\eta : E_{\text{eff}} \mapsto R^{(3)} := \frac{16\pi G}{c^4} E_{\text{eff}}. \tag{14}$$

Remark 7.1 (Critical Clarification). Using Eq. (14) for microscale estimates does *not* assert that gravity fails to back-react. It asserts that, for the micro configurations under study, the dimensionless gravitational compactness is so small that solving the full coupled field equations produces only perturbatively tiny changes in equilibrium geometry.

7.3 Micro Non-Collapse from Compactness Scaling

Proposition 7.1 (Compactness Argument)**.** For a localized object of characteristic size R and total mass-energy M, strong-field collapse requires compactness

$$\epsilon := \frac{2GM}{Rc^2} \gtrsim 1. \tag{15}$$

For hadronic scales $M \sim 1\,\text{GeV}/c^2$ and $R \sim 1\,\text{fm}$, one finds $\epsilon \ll 1$ by many orders of magnitude. Therefore, strong-field backreaction and gravitational crushing are absent without modifying GR.

Proof. For a proton, $M \approx 1.67 \times 10^{-27}$ kg and $R \approx 10^{-15}$ m. The Schwarzschild radius is $r_s = 2GM/c^2 \approx 2.5 \times 10^{-54}$ m, giving $\epsilon \approx 10^{-39}$. This is so small that gravitational corrections to the equilibrium structure are utterly negligible compared to the strong nuclear force that actually determines hadronic structure. ☐

Remark 7.2 (Hierarchy Not Solved by Fiat). The absence of micro collapse is not achieved by "turning off" gravity; it follows from the smallness of G and the small total mass-energy M. However, *deriving* G from internal geometric moduli remains a separate completion obligation (Section 11).

7.4 Equivalence Principle: When It Holds and Violations

Postulate 7.1 (Effective Motion Law for Defects)**.** Defects are extended objects with internal torsion coupling. In the point-particle limit with negligible torsion gradients and averaged spin polarization, the defect worldline follows an effective geodesic principle. Corrections appear as spin/chirality-dependent forces:

$$\frac{Du^\mu}{d\tau} = f^\mu_{\text{tors}}(S, \nabla S; \text{polarization, chirality}) + O(\text{multipoles}), \tag{16}$$

implying controlled equivalence-principle violations in polarized/chiral sectors while preserving standard universality in macroscopic unpolarized matter.

This is consistent with the Mathisson–Papapetrou–Dixon equations for spinning particles [47, 48] and their extension to Einstein–Cartan theory [10].

8 Interactions as Cobordisms and the Y^4 Nexus

Definition 8.1 (Cobordism Interaction)**.** An *interaction* is a compact 4-manifold-with-boundary X that cobords incoming and outgoing worldtubes:

$$\partial X = \bigsqcup_{a=1}^{n_{\text{in}}} W_a^{\text{in}} \sqcup \bigsqcup_{b=1}^{n_{\text{out}}} W_b^{\text{out}}. \tag{17}$$

This definition draws on the mathematical theory of cobordism [20, 21] and its role in topological quantum field theory [18]. In TQFT, a cobordism between manifolds induces a linear map between associated Hilbert spaces [19].

Postulate 8.1 (Y^4 Nexus)**.** The canonical tripartite junction is a 4-manifold Y^4 with boundary

$$\partial Y^4 = W_1 \sqcup W_2 \sqcup W_3, \tag{18}$$

and boundary/junction consistency conditions enforce a $\mathbb{Z}_3$ constraint on holonomy data (denoted $z_i \in \mathbb{Z}_3$):

$$z_1 z_2 z_3 = 1. \tag{19}$$

The Y^4 nexus is the geometric realization of a three-particle vertex. The $\mathbb{Z}_3$ constraint encodes the requirement that color charges combine to form a singlet, mirroring the structure of baryon vertices in QCD.

9 Gauge-Group Emergence: Obstruction-Theoretic Program

9.1 Bundles on Defect Complements

Construction 9.1 (Complement-Bundle Data)**.** Let $U := \mathcal{M} \setminus W$ be the complement of a defect worldtube. Assign to U a principal G-bundle $P \to U$ with connection A representing emergent gauge data. Holonomies around nontrivial cycles in U define group elements constrained by the defect invariants $\mathcal{I}(W)$.

This construction is motivated by the Aharonov–Bohm effect [50] and the general principle that gauge fields can be characterized by their holonomies around non-contractible loops.

9.2 Extension Across Junctions as an Obstruction Problem

Construction 9.2 (Junction Extension Problem)**.** Given three complements $U_i = \mathcal{M} \setminus W_i$ with bundles (P_i, A_i), a junction cobordism Y^4 defines a gluing domain whose boundary data must extend to a bundle on Y^4. The obstruction to extension lives in cohomology with coefficients in $\pi_k(G)$ and/or the center $Z(G)$, depending on the chosen Postnikov tower.

The mathematical framework for such obstruction problems is developed in algebraic topology [51, 52].

Conjecture 9.1 (Minimal-Group Emergence)**.** Under: (i) Pin^+ admissibility, (ii) existence of $\mathbb{Z}_3$-center junction constraints (19), and (iii) compatibility with a $\mathbb{Z}_2$ non-orientable (fermionic) sector, the minimal compact gauge structure enabling consistent extension across all admissible defect cobordisms is

$$G_{\text{eff}} \simeq (\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1))/\mathbb{Z}_6, \tag{20}$$

with representation content determined by $\mathcal{I}(W)$.

Remark 9.1 (What Must Be Proven). A rigorous proof requires:

(i) specifying the exact cobordism category of admissible defects and junctions,

(ii) specifying the bundle data and equivalence (gauge) relations,

(iii) computing the obstruction classes for extension across Y^4 and showing uniqueness/minimality of the resulting G_{eff}.

This program is inspired by work on anomaly cancellation and global gauge structure in the Standard Model [53, 54].

10 Quantization: Defect Path Integral and TQFT

10.1 Why Quantization is Required

Defect topology alone defines states but not quantum superposition or scattering amplitudes. A monist theory must supply a measure over geometries and defect embeddings that yields interference and cross-sections. The framework of topological quantum field theory [18, 19] provides the natural mathematical language.

10.2 Defect-Cobordism Category and Quantization Functor

Definition 10.1 (Defect Cobordism Category)**.** Let $\mathbf{Cob}_{\text{def}}$ be a category whose *objects* are 3D "asymptotic" data sets encoding defect boundaries and invariant packages, and whose *morphisms* are 4D cobordisms X equipped with admissible (g, Γ, S) data satisfying boundary conditions.

Postulate 10.1 (Quantization as a Functor)**.** Quantization is a symmetric monoidal functor

$$Z : \mathbf{Cob}_{\text{def}} \to \mathbf{Hilb}, \tag{21}$$

assigning a Hilbert space to each asymptotic defect boundary object and a linear map (amplitude) to each cobordism.

This definition follows Atiyah's axiomatization of TQFT [18], extended to include defect structures. The cobordism hypothesis of Baez–Dolan and Lurie [55, 56] provides a framework for classifying such functors.

10.3 Path Integral Realization

Postulate 10.2 (Defect Path Integral Amplitude)**.** For asymptotic in/out defect sectors, define the transition amplitude by

$$\mathcal{A}(W_{\text{in}} \to W_{\text{out}}) := \sum_{[X]\in\mathcal{C}(W_{\text{in}},W_{\text{out}})} \frac{1}{|\text{Aut}(X)|} \int \frac{\mathcal{D}g\,\mathcal{D}\Gamma\,\mathcal{D}S}{\text{Diff}(\mathcal{M})} \exp\left(\frac{i}{\hbar} S_{\text{bulk}}[g, \Gamma, S]\right), \tag{22}$$

where the sum runs over admissible defect cobordisms X between the boundary sectors (including junctions), and the quotient indicates gauge-fixing/modding out diffeomorphisms.

Remark 10.1 (Completion Requirement). Equation (22) is a definition template. A complete theory must specify: (i) the admissible class $\mathcal{C}$ of cobordisms/embeddings, (ii) the measure and gauge-fixing procedure, and (iii) the semiclassical saddle structure reproducing known scattering limits.

11 Constants and Couplings as Derived Moduli

11.1 Principle: Couplings Must Arise from the Action

Principle 11.1 (Coupling Derivation Obligation)**.** Any claim that α (the fine-structure constant) or G (Newton's constant) is "derived" must exhibit a chain:

(i) identify a dimensionless geometric/topological functional $\mathcal{I}$ of (g, Γ, S) and defect/junction data,

(ii) show that integrating out microstructure yields an effective action with coefficients determined by $\mathcal{I}$,

(iii) specify the renormalization convention and scale at which the derived coefficient matches the observed coupling.

This principle guards against numerological claims by demanding a complete derivation within the theoretical framework.

11.2 Effective Gauge Kinetic Terms from Nexus Geometry

Construction 11.1 (Emergent Gauge Kinetic Coefficients)**.** Assume that in the complement of defects/junctions, coarse-graining produces effective gauge kinetic terms:

$$S_{\text{eff}} \supset -\frac{1}{4}\int d^4x\sqrt{-g}\left[\frac{1}{g_1^2}F^{(1)}_{\mu\nu}F^{(1)\mu\nu} + \frac{1}{g_2^2}\mathrm{Tr}(F^{(2)}_{\mu\nu}F^{(2)\mu\nu}) + \frac{1}{g_3^2}\mathrm{Tr}(F^{(3)}_{\mu\nu}F^{(3)\mu\nu})\right], \tag{23}$$

where the g_i are determined by geometric functionals of nexus/defect microstructure:

$$\frac{1}{g_i^2} = \mathcal{I}_i[g, \Gamma, S; \text{defects/junctions}]_{\mu_{\text{UV}}} \quad (\text{bare at scale } \mu_{\text{UV}}). \tag{24}$$

Remark 11.1 (Numerology vs. Derivation). Any numeric coincidence (such as $\alpha^{-1} \approx 137$) can be kept only as a candidate leading-order topological estimate after the normalization and RG flow are fixed by the action-based derivation above. Until then, it is not a theory prediction.

11.3 Gravitational Coupling from Torsion Stiffness

Construction 11.2 (Planck Scale from Torsion Parameters)**.** A monist mechanism for G must relate the effective Einstein–Hilbert coefficient to torsion-sector microphysics. At the level of dimensional analysis and effective matching:

$$\frac{1}{\kappa_{\text{eff}}} \sim M^2_{\text{Pl,eff}} \sim \beta S_0^2 \times \mathcal{J}[\text{topology}], \tag{25}$$

where $\mathcal{J}$ is a dimensionless geometric/topological functional determined by admissible defect/junction sectors.

Remark 11.2 (Completion Requirement). The theory must *compute* $\mathcal{J}$ (not assume it) and then run the resulting effective couplings to laboratory scales using the renormalization group.

12 Cosmology with Torsion and Singularity Avoidance

Postulate 12.1 (Effective Modified Friedmann Equation)**.** At the homogeneous/isotropic effective level:

$$H^2 = \frac{8\pi G}{3}\rho_{\text{tot}} - \frac{\rho^2_{\text{tot}}}{\rho_{\text{crit}}} + \frac{V(S)}{3}, \tag{26}$$

where the $-\rho^2$ term encodes torsion/spin-induced repulsion at high density and $V(S)$ is induced by the torsion potential.

This modified Friedmann equation is characteristic of Einstein–Cartan cosmology [45, 46]. The $-\rho^2$ term generates a bounce at high densities, replacing the Big Bang singularity with a minimal but finite scale factor. This is the cosmological expression of the "counter-curvature" channel: torsion contributions generate repulsive effects that can prevent singularities [57].

13 Predictions (Parameter-Dependent and Constrainable)

The following predictions are stated as functions of the theory parameters (β, λ, S_0) and defect boundary conditions. They become testable once these parameters are either derived or constrained by other observations.

Postulate 13.1 (Gravitational Wave Echoes)**.** Near-horizon torsion structure can produce ringdown echoes with delay scaling

$$\Delta t_{\text{echo}} \propto \ln\left(\frac{R_{\text{EH}}}{\ell_{\text{micro}}}\right), \tag{27}$$

where R_{EH} is the event horizon radius and ℓ_{micro} is the torsion microstructure scale. A complete prediction must provide the echo amplitude spectrum as a function of (β, λ, S_0) and defect boundary conditions.

Postulate 13.2 (Chiral/Spin-Dependent Gravity)**.** Torsion-locking couplings induce controlled equivalence-principle violations in polarized/chiral matter:

$$\Delta a \sim \epsilon_{\text{EP}}\, g, \quad \epsilon_{\text{EP}} = \epsilon_{\text{EP}}(\beta, \lambda, S_0; \text{polarization}). \tag{28}$$

Postulate 13.3 (Nexus Resonances)**.** High-energy collisions may excite nexus geometry, producing resonances whose mass and rate are parameter-dependent. A viable prediction must be expressed as $\sigma(pp \to X) \times \text{BR}(X \to \gamma\gamma, VV)$ with uncertainties.

14 Completion Checklist

To close the theory, the following constructions are mandatory:

1. **Quantization**: Specify the defect cobordism category $\mathbf{Cob}_{\text{def}}$, the measure and gauge-fixing in Eq. (22), and show recovery of quantum interference and known scattering limits.
2. **Constants**: Derive the effective couplings g_i and κ_{eff} from coarse-graining of S_{bulk} with defect/junction microstructure, including renormalization conventions and running.
3. **Gauge Group Proof**: Implement the obstruction-theory program and prove minimality/uniqueness of G_{eff} for the admissible junction class.

These represent the minimal requirements for the theory to be considered "closed" in the sense of making definite, falsifiable predictions.

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References

[1] J. A. Wheeler, "On the nature of quantum geometrodynamics," *Ann. Phys.* **2**, 604–614 (1957).

[2] J. A. Wheeler, *Geometrodynamics* (Academic Press, New York, 1963).

[3] C. W. Misner and J. A. Wheeler, "Classical physics as geometry," *Ann. Phys.* **2**, 525–603 (1957).

[4] J. A. Wheeler, "Geons," *Phys. Rev.* **97**, 511–536 (1955).

[5] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (W. H. Freeman, San Francisco, 1973).

[6] É. Cartan, "Sur une généralisation de la notion de courbure de Riemann et les espaces à torsion," *C. R. Acad. Sci. (Paris)* **174**, 593–595 (1922).

[7] É. Cartan, "Sur les variétés à connexion affine et la théorie de la relativité généralisée," *Ann. Éc. Norm.* **40**, 325–412 (1923); **41**, 1–25 (1924); **42**, 17–88 (1925).

[8] D. W. Sciama, "The physical structure of general relativity," *Rev. Mod. Phys.* **36**, 463–469 (1964).

[9] T. W. B. Kibble, "Lorentz invariance and the gravitational field," *J. Math. Phys.* **2**, 212–221 (1961).

[10] F. W. Hehl, P. von der Heyde, G. D. Kerlick, and J. M. Nester, "General relativity with spin and torsion: Foundations and prospects," *Rev. Mod. Phys.* **48**, 393–416 (1976).

[11] T. W. B. Kibble, "Topology of cosmic domains and strings," *J. Phys. A* **9**, 1387–1398 (1976).

[12] A. Vilenkin, "Cosmic strings and domain walls," *Phys. Rep.* **121**, 263–315 (1985).

[13] A. Vilenkin and E. P. S. Shellard, *Cosmic Strings and Other Topological Defects* (Cambridge University Press, Cambridge, 2000).

[14] T. H. R. Skyrme, "A non-linear field theory," *Proc. Roy. Soc. London A* **260**, 127–138 (1961).

[15] T. H. R. Skyrme, "A unified field theory of mesons and baryons," *Nucl. Phys.* **31**, 556–569 (1962).

[16] G. S. Adkins, C. R. Nappi, and E. Witten, "Static properties of nucleons in the Skyrme model," *Nucl. Phys. B* **228**, 552–566 (1983).

[17] E. Witten, "Current algebra, baryons, and quark confinement," *Nucl. Phys. B* **223**, 433–444 (1983).

[18] M. F. Atiyah, "Topological quantum field theories," *Publ. Math. IHÉS* **68**, 175–186 (1988).

[19] E. Witten, "Topological quantum field theory," *Commun. Math. Phys.* **117**, 353–386 (1988).

[20] R. Thom, "Quelques propriétés globales des variétés différentiables," *Comment. Math. Helv.* **28**, 17–86 (1954).

[21] J. W. Milnor and J. D. Stasheff, *Characteristic Classes* (Princeton University Press, Princeton, 1974).

[22] R. C. Kirby and L. R. Taylor, “Pin structures on low-dimensional manifolds,” in *Geometry of Low-Dimensional Manifolds*, vol. 2, London Math. Soc. Lecture Note Series **151** (Cambridge University Press, 1990), pp. 177–242.

[23] H. B. Lawson and M.-L. Michelsohn, *Spin Geometry* (Princeton University Press, Princeton, 1989).

[24] A. Trautman, “Einstein–Cartan theory,” in *Encyclopedia of Mathematical Physics*, eds. J.-P. Françoise, G. L. Naber, and T. S. Tsun (Elsevier, Oxford, 2006), vol. 2, pp. 189–195.

[25] A. Palatini, “Deduzione invariantiva delle equazioni gravitazionali dal principio di Hamilton,” *Rend. Circ. Mat. Palermo* **43**, 203–212 (1919).

[26] J. A. Schouten, *Ricci-Calculus*, 2nd ed. (Springer, Berlin, 1954).

[27] M. F. Atiyah, R. Bott, and A. Shapiro, “Clifford modules,” *Topology* **3**, 3–38 (1964).

[28] T. Kaluza, “Zum Unitätsproblem der Physik,” *Sitzungsber. Preuss. Akad. Wiss. Berlin (Math. Phys.)* **1921**, 966–972 (1921).

[29] O. Klein, “Quantentheorie und fünfdimensionale Relativitätstheorie,” *Z. Phys.* **37**, 895–906 (1926).

[30] A. Einstein, “Einheitliche Feldtheorie von Gravitation und Elektrizität,” *Sitzungsber. Preuss. Akad. Wiss.* **22**, 414–419 (1925).

[31] H. F. M. Goenner, “On the history of unified field theories,” *Living Rev. Relativ.* **7**, 2 (2004).

[32] M. B. Hindmarsh and T. W. B. Kibble, “Cosmic strings,” *Rep. Prog. Phys.* **58**, 477–562 (1995).

[33] N. D. Mermin, “The topological theory of defects in ordered media,” *Rev. Mod. Phys.* **51**, 591–648 (1979).

[34] G. E. Volovik, *The Universe in a Helium Droplet* (Oxford University Press, Oxford, 2003).

[35] F. W. Hehl and B. K. Datta, “Nonlinear spinor equation and asymmetric connection in general relativity,” *J. Math. Phys.* **12**, 1334–1339 (1971).

[36] H. B. Nielsen and P. Olesen, “Vortex-line models for dual strings,” *Nucl. Phys. B* **61**, 45–61 (1973).

[37] S. Coleman, “Fate of the false vacuum: Semiclassical theory,” *Phys. Rev. D* **15**, 2929–2936 (1977).

[38] S. Weinberg, “The cosmological constant problem,” *Rev. Mod. Phys.* **61**, 1–23 (1989).

[39] I. L. Shapiro, “Physical aspects of the space-time torsion,” *Phys. Rep.* **357**, 113–213 (2002).

[40] E. Witten, “Baryons in the $1/N$ expansion,” *Nucl. Phys. B* **160**, 57–115 (1979).

[41] S. Weinberg, “Phenomenological Lagrangians,” *Physica A* **96**, 327–340 (1979).

[42] A. M. Polyakov, *Gauge Fields and Strings* (Harwood Academic Publishers, Chur, 1987).

[43] W. Helfrich, “Elastic properties of lipid bilayers: theory and possible experiments,” *Z. Naturforsch. C* **28**, 693–703 (1973).

[44] P. B. Canham, “The minimum energy of bending as a possible explanation of the biconcave shape of the human red blood cell,” *J. Theor. Biol.* **26**, 61–81 (1970).

[45] N. J. Popławski, "Nonsingular Dirac particles in spacetime with torsion," *Phys. Lett. B* **690**, 73–77 (2010).

[46] N. J. Popławski, "Cosmology with torsion: An alternative to cosmic inflation," *Phys. Lett. B* **694**, 181–185 (2010).

[47] A. Papapetrou, "Spinning test-particles in general relativity. I," *Proc. Roy. Soc. London A* **209**, 248–258 (1951).

[48] W. G. Dixon, "Dynamics of extended bodies in general relativity. I. Momentum and angular momentum," *Proc. Roy. Soc. London A* **314**, 499–527 (1970).

[49] R. T. Hammond, "Torsion gravity," *Rep. Prog. Phys.* **65**, 599–649 (2002).

[50] Y. Aharonov and D. Bohm, "Significance of electromagnetic potentials in the quantum theory," *Phys. Rev.* **115**, 485–491 (1959).

[51] N. Steenrod, *The Topology of Fibre Bundles* (Princeton University Press, Princeton, 1951).

[52] A. Hatcher, *Algebraic Topology* (Cambridge University Press, Cambridge, 2002).

[53] D. Tong, "Line operators in the Standard Model," *J. High Energy Phys.* **2017**(07), 104 (2017).

[54] J. Wang and X.-G. Wen, "A non-perturbative definition of the Standard Model," *Phys. Rev. Research* **2**, 023356 (2020); arXiv:1809.11171.

[55] J. C. Baez and J. Dolan, "Higher-dimensional algebra and topological quantum field theory," *J. Math. Phys.* **36**, 6073–6105 (1995).

[56] J. Lurie, "On the classification of topological field theories," in *Current Developments in Mathematics 2008* (International Press, 2009), pp. 129–280; arXiv:0905.0465.

[57] A. Trautman, "Spin and torsion may avert gravitational singularities," *Nature Phys. Sci.* **242**, 7–8 (1973).

[58] P. A. M. Dirac, "An extensible model of the electron," *Proc. Roy. Soc. London A* **268**, 57–67 (1962).

[59] Y. Nambu, "Duality and hadrodynamics," Lecture notes prepared for Copenhagen Summer Symposium (1970).

[60] T. Goto, "Relativistic quantum mechanics of one-dimensional mechanical continuum and subsidiary condition of dual resonance model," *Prog. Theor. Phys.* **46**, 1560–1569 (1971).